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Numerical simulations in the assets administration of pension funds: a scoping review

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Resumen

Los planes de pensiones enfrentan desafíos no vistos en décadas en la economía global (Mercer, 2022). Es imperativo conocer metodologías cuantitativas robustas que equilibren los activos y pasivos de los fondos logrando viabilidad financiera. En este trabajo se realiza una revisión de alcance para responder a la pregunta ¿Qué se sabe de la literatura existente sobre simulaciones numéricas en la administración de activos de los fondos de pensiones que aseguren la viabilidad financiera y la sostenibilidad? Los criterios de elegibilidad fueron: artículos revistas indizadas dentro del período de 2000 a 2025, escritos en inglés o español y con métodos cuantitativos. Las herramientas de IA Elicit, Connected Papers y Google Scholar son las fuentes de búsqueda específicas utilizadas. Los resultados son que, de un total de 64 artículos, 46 se identificaron en el área de pensiones, varios métodos numéricos en su mayoría con métodos estocásticos, pocos estudios con resultados numéricos.

Palabras clave: pensiones, administración de activos, métodos numéricos, revisión de alcance.

Abstract

Pension plans face challenges not seen in decades in the global economy (Mercer, 2022). It is imperative to know robust quantitative methodologies that balance assets and liabilities of pension funds, achieving financial viability. This work conducted a scoping review to answer the question: *What is known from the existing literature about numerical simulations in the asset administration of pension funds that ensure financial viability and sustainability?* The eligibility criteria were: peer-reviewed journal papers within the period of 2000 to 2025, written in English or Spanish, and with quantitative methods. The AI tools Elicit; Connected Papers, and Google Scholar are the specific searching sources used. The results are that, from a total of 64 papers, 46 are related to the pensions area, several numerical methods, of which the majority mention stochastic methods, and only a few produce numerical results.

Keywords: pensions, asset administration, numerical methods, scoping review.

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Introduction

Pensions are a major concern due to the increasing burden that these place on state plans and the financial pressure placed on employers (Stewart, 1983). This problem is exacerbated by an aging population, which creates fiscal challenges in many countries (Barr, 2006). Defined benefit pension plans in the United Kingdom, for example, particularly those in deficit are an area of particular concern, with potential knock-on effects for employers and plan administrators (Lomax, 2012). The complexity of pension plans, along with economic and equity issues, have also increased over time (Haberman, 1985).

Stewart (1983) noted a long time ago that no matter how you look at it, pensions are a problem. It is mentioned that the Institute for Policy Studies expressed concern more than 40 years ago about the increased burden on state pensions that would be seen in 50 years (i.e., about 10 years from now), when the number of pensioners increased relative to the number of taxpayers. The Institute for Fiscal Studies also expressed similar concerns and criticized the fact that the long-term cost of pensions was not sufficiently considered when the state scheme was expanded in 1978 to provide an additional income-related pension. At that time, employers, who ultimately bear the financial burden, were hard-pressed to fund their employees' accrued pension entitlements in defined-benefit pension schemes tied to final salary and were unable to make the additional contributions necessary to preserve the value of current workers' pensions and the deferred pensions of former employees.

Haberman (1985) reported that from the very beginning of the formation of pension plans in the United Kingdom; actuaries have been involved in their design, investment, valuation, and solvency. He mentioned that if a study had been conducted 40 years before that time, it would show how the systems had become more complicated and comprehensive, how economic and equity issues had increased and how the actuarial profession had successfully adapted its theory and practice to accommodate change. However, he mentioned that the rapid growth of occupational pension plans raised concerns about the overall effect on the national economy.

More recently, Bejaković and Šonje (2023) mentioned that a pension system in modern society has several functions, the main ones being the equitable distribution of income among individuals and families throughout their lives, the promotion of individual and national savings, and the reduction of poverty during old age and labor inactivity. One of the most important objectives of pension insurance is to maintain the income older people had during their working lives. While older people were more likely to belong to the relatively poor, current indicators point to quite positive effects, i.e. a significant reduction in poverty among older people.

Numerous studies have been conducted on the importance of researching various aspects of pensions worldwide. Barr (2006) mentions that effective governance and economic growth are of utmost importance in addressing pension funding challenges. He also points out that the debate between pay-as-you-go and funding is considered secondary to the importance of good governance and economic development. He concludes that, while there is a problem in pension funding due to population aging, he does not classify it as a crisis.

Desmond (2012) notes that the issue with pensions is finding an adequate balance between ensuring people that their financial contributions will be protected and that this will result in a reasonable pension. This involves protecting against financial catastrophes that may occur over time. The author also notes the growing importance attached to the issue of pensions among the media, professionals, governments, and in international regulations.

According to Mercer (2022), pension plans are facing challenges not seen in decades in the global economy. This is because these plans have been affected by events such as the pandemic, global conflicts, supply chain disruptions, and a reversal of the trend toward globalization. This has led to higher levels of inflation, rising interest rates, and greater uncertainty in economic conditions; all this while life expectancy continues to increase and populations are aging. Therefore, these negative events occur at a time when individuals are taking on more responsibility for obtaining their pensions, as defined contribution (DC) plans become more prevalent, and their development and creation continue to increase worldwide. The mandatory pension or social security plan in Mexico is among the worst rated in the world with a score of 56.1 out of 100. Compared to other countries, it is far below those of Denmark (82 points) or Uruguay (71.5 points) in Latin America. The main problem in Mexico is that its system has significant risks and deficiencies that must be addressed, and without these improvements, its effectiveness and long-term sustainability could be questioned.

To counteract these negative aspects of pensions, various authors have long explored the modeling of pension fund management. Anderson (2002) and Haberman (2004) emphasize the importance of using stochastic modeling to reduce the variance and risk of managing these funds. Fabozzi et al. (2005) agree with these studies, highlighting the everyday use of stochastic methods in modeling defined-benefit pension funds, particularly in the Netherlands. These authors model pension fund management using stochastic methods (Anderson, 2002; Haberman, 2004; Fabozzi et al., 2005; Khorasanee, 1997).

Stochastic methods focus on the types of instruments in which a pension fund should invest, use time series methods to model rates of return, and analyze fund behavior in the face of investment shocks. For example, Anderson (2002) conducts a stochastic modeling exercise of assets and liabilities for a pension fund investment portfolio, illustrating the reduction in the variance of expected

future returns achieved when the portfolio is invested in indexed securities. He also mentions that investing in equity assets increases the value of returns, but that the variance also increases, with a significant probability that the assets will not be sufficient to cover the pension fund's liabilities.

Haberman (2004), for his part, describes a mathematical model that analyzes the behavior of defined benefit pension funds with stochastic investment returns. The variables focused on in his model are amortization periods, the time over which adjustments are made to the value of contributions, the frequency of actuarial valuation, and the choice of funding method. The models used to model stochastic rates of return are time series, primarily autoregressive.

Fabozzi et al. (2005) analyze pension fund modeling from the perspective of fund management. Their study is based on interviews with administrators, regulators, consultants, and academics from four different countries (the Netherlands, Switzerland, the United Kingdom, and the United States). The objective was to understand the role that modeling plays in defined benefit pension funds from the perspective of market participants. The findings of this study are that modeling is considered an indispensable tool by many participants. The need to manage the risk inherent in defined benefit pension plans is the primary motivation behind the growing use of modeling.

The studies proposed by these authors refer on how to determine the optimal approach to asset management in a pension fund; however, it is unclear whether these methodologies have been explored in pension funds to ensure financial viability and sustainability. For these reasons, a scoping review is conducted in this work to systematically map the research in this area and identify any existing gaps in knowledge. Therefore, the following question is formulated: *What is known from the existing literature about numerical simulations in the assets administration of pension funds that ensure financial viability and sustainability?*

Methods

The methods employed to answer the question previously defined in this work are a scoping review conducted under the PRISMA ScR methodology. The scoping review is recommended when a study wishes to generate a robust literature review, validated to complement research work or to generate new research areas with a scientific approach (Arksey and O'Malley, 2005; Tricco et al., 2018; Codina, 2024). These kinds of studies allow for a detailed description of the research findings from other authors and to have a summary of these findings for others who might be interested or to undertake the work themselves (Antman et al. 1992, taken from Arksey & O'Malley, 2005).

The main objective in this work is to identify research gaps in existing literature by systematically mapping the research previously done. As Tricco et al. (2018) mention, "*This type of scoping study takes the process of dissemination one step further by drawing conclusions from*

existing literature regarding the overall state of research activity. Specifically designed to identify gaps in the evidence base where no research has been conducted, the study may also summarise and disseminate research findings as well as determine the relevance of a full systematic review in specific areas of inquiry. The process under the PRISMA Extension for Scoping Reviews (PRISMA ScR) consists in defining several elements which are mentioned as follows.

Eligibility criteria

The papers reviewed followed a specific criteria. Peer-reviewed journal papers were included within the period of 2000 to 2025, written in English or Spanish and including quantitative methods only. The period is defined to search for numerical methods from the last 25 years, which is believed to be a reasonable time frame, and a quantitative method is required to analyze numerical simulation results. Papers were excluded if they did not fit into the conceptual framework of the study, which was defined as numerical methods to simulate rates of return preferably related to the pension fund area; however, the eligibility criteria are not limited to the pension area only, the search is extended also to include finance studies.

Information sources

The website "Google Scholar" was used as a first approach. This website uses an algorithm that searches within scholarly publishers, universities, and professional societies, and it indexes articles, theses, books, abstracts, and court opinions (OpenAI, 2025a).

The IA Elicit platform was also used to consult databases from January 1995 to June 2025. Elicit is an artificial intelligence tool that assists in the search, analysis, and synthesis of scientific information, accessing a database of over 126 million academic articles from the Semantic Scholar corpus, which covers all academic disciplines (Arroyo-Machado, 2024). When the search is performed, only the papers that meet the eligibility criteria are included in the scoping review of this work.

Additionally, the IA Connected Papers was used as a second tool, a visual aid to help researchers and scientists find and explore documents relevant to their field of work. It selects documents with the strongest connections to the source document, using the database, the Semantic Scholar Paper Corpus (Murillo González & López, 2025).

Search strategy

The search was conducted through scholar.google.com with the specific search: numerical simulation of rates of return in the assets administration of pension funds. The advanced search was selected to find the papers that met the eligibility criteria.

On a second approach, as elicit.com searches through questions, the following questions related to the objective of this work are introduced, and selecting the option “find papers”: How do stochastic rates of return perform in pension funds? The tables generated by elicit.com are analyzed one entry at a time. The papers that do not meet the eligibility criteria are discarded.

The additional tool Connected Papers is used as follows: on the main page of the website connectedpapers.com, a keyword is introduced (i.e., Numerical Simulation of Pension Fund Returns), the IA shows different papers related to this topic, and the user has to select a paper that is most related to the topic.

Data items

The papers on Google Scholar were selected manually, with each abstract reviewed individually. The papers that met the eligibility criteria were used to generate the results in this work.

The Elicit algorithm determines whether the PDF of the articles, related to the search and the eligibility criteria defined previously, is available. When available, direct access is provided by clicking on the title; however, when restricted-access articles are present, the system utilizes the abstract to generate answers or analyses (Universo Abierto, 2025).

On the other hand, the Connected Papers algorithm selects the most relevant documents related to the document title, DOI, PubMed, or ArXiv number of a source document; it then generates a graph with all documents related to the initial document, and the documents are sorted by similarity. The algorithm selects documents that do not even directly cite each other but can be strongly connected and highly ranked (Universo Abierto, 2020). The similarity metric is based on the concepts of Co-citation and Bibliographic Coupling. According to this measure, two papers that have highly overlapping citations and references are presumed to have a higher chance of treating a related subject matter (Connected Papers, s.f.). Once the graph is generated, the papers can be selected manually directly from the graph, which are analyzed in their content, and ultimately, the analysis is performed.

Synthesis of results

The elicit.com platform transforms the question into a topic relevant to it. The resulting topic is the following search: Stochastic Returns in Pension Funds. Then, the documents are selected manually according to the eligibility criteria.

The platform connectedpapers.com displays several peer-reviewed papers for the user to select the most relevant to the introduced topic. The paper chosen was Chang et al. (2019), as it was deemed the most appropriate. The AI platform then displays connected documents to the selected one. Then, the results presented in this work are selected manually based on the eligibility criteria.

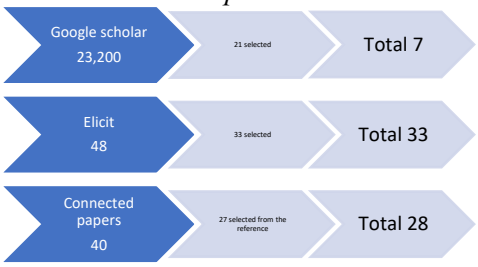
Once the pool of papers is selected based on the steps taken before, it is an important part of the process that the papers are considered only if they could potentially meet our eligibility criteria. Therefore, the documents selected are manually reviewed to discard any duplicated papers or documents whose topics do not relate to the objective of this work. With this final selection, the IA Mapify is going to be used to create a diagram with the most relevant topics visually organized. The following section shows the results of performing the method described.

Results

To reach the objective of this work, defined as a research question: What is known from the existing literature about numerical simulations in the assets administration of pension funds that ensure financial viability and sustainability?

And following the methods previously described in this work, Figure 1 shows the number of citations screened, detailing the duplicates removed and full-text documents assessed.

Figure 1.
Full-text documents selection process to meet the eligibility criteria.



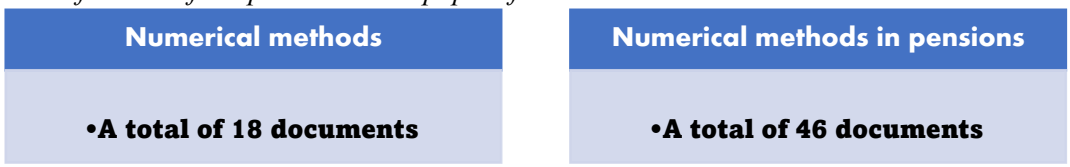
Source: author's elaboration.

As previously defined, three databases were consulted. When searching on Google scholar a total of 23,200 results are found; from these 19,600 results for papers are selected to meet with the period 2000 to 2025; 6,460 results are chosen for peer-reviewed journals; 30 results for the papers with the most relevant to the topic from which 21 results are selected to have a pdf available and finally 7 papers selected that met the eligibility criteria. When searching on Elicit, a total of 48 most relevant papers are found, of which 33 papers are selected that meet the eligibility criteria. When searching on Connected papers, a total of 40 papers were found to be connected to Chang, et al. (2019); from which 27 papers met the eligibility criteria, giving a total of 28, as it includes the 1 reference paper. In total, 69 peer-reviewed papers met the eligibility criteria; however, after finding duplicated papers, a total of 64 papers were finally included for the analysis of this work.

The results show that when searching peer-reviewed papers that meet the defined eligibility criteria, and with the objective to answer the question W*hat is known from the existing literature*

about the numerical simulation of rates of return in the assets administration of pension funds that ensure financial viability and sustainability? A total of 64 papers were found as the most relevant and most connected to the topic defined in this work. Figure 2 shows the classification into two groups: the papers related to numerical models to simulate rates of return and the ones related to numerical models to simulate rates associated with pension funds.

Figure 2.
Classification of the peer-reviewed papers found.



Source: author's elaboration.

Although our objective in this work is to find what is known of this topic in the literature related to the pensions area, 18 documents can be added to our analysis, even if these are linked to the area of finance. Just a reminder at this point is that the reason to find relatively more papers in the pensions area is because the AI connectedpapers.com is a useful tool to show papers connected to the one chosen by the user, as was done in this work. Figure 3 shows the diagram of a total of 40 papers directly connected to "stochastic interest rate and mean-reverting returns", as seen in the middle of the diagram. As mentioned before, a total of 28 from these 40 were selected.

A more detailed description of the total 64 peer-reviewed papers selected is shown in Tables 1 and 2. The characteristics shown are useful for performing an analysis to answer the question related to the objective of this work.

Table 1.
Study objectives from a total of 64 documents selected.
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Title of the study	Authors	Objectives
A Long-Term Perspective for Pension Fund Portfolios in Chile, Peru, and Mexico.	David Tuesta, Carlos Herrera (2014)	- Review pension fund investment returns in Chile, Peru, and Mexico with a long-term perspective. - Analyze historical returns of private pension systems. - Evaluate potential returns from new multi-fund schemes using Monte Carlo simulations.
A quantitative model for the asset liability management of a Pension Fund.	Giuseppina Cannas (2011)	- Propose a quantitative method to determine the optimal investment policy of a pension fund under financial and actuarial risks. - Develop an original model for optimal financial investment policy considering financial and actuarial risks. - Ensure pension plan sustainability by balancing active and retired members.
An Investment and Consumption Problem with CIR Interest Rate and Stochastic Volatility.	Hao Chang, X. Rong (2013)	- To study optimal investment and consumption strategies under stochastic interest rate and stochastic volatility models. - To maximize the expected discounted utility of consumption and terminal wealth. - To solve nonlinear second-order partial differential equations to obtain closed-form solutions.

An optimal investment strategy for DC pension plans with costs and the return of premium clauses under the CEV model.	Xiaoyi Tang, Wei Liu, Wanyin Wu, Yijun Hu (2024)	- To present a novel optimization model for DC pension plans with return of premium clauses. - To incorporate trading fees and taxes into the model. - To find the optimal investment strategy that maximizes the expected utility of the terminal wealth.
Asset allocation for a DC pension fund with stochastic income and mortality risk: A multi-period mean–variance framework.	H. Yao, Yongzeng Lai, Q. Ma, Minjie Jian (2014)	- Develop a multi-period mean–variance framework for asset allocation in a DC pension fund. - Incorporate stochastic income and mortality risk into the framework.
Asset/Liability Management for Pension Funds Using CVaR Constraints.	Bogentoft, Erik, Edwin Romeijn, H., Uryasev, Stanislav (2001)	- To study formal optimal decision approaches for a multi-period Asset/Liability Management model for a pension fund. - To apply Conditional Value-at-Risk (CVaR) as a risk measure in this context. - To develop and apply a model based on sample-path simulation and optimization using linear programming.
Bayesian dynamic factor models and variance matrix dis-counting for portfolio allocation.	O. Aguilar, M. West (2000)	- Develop dynamic factor models for multivariate financial time series. - Incorporate stochastic volatility components for latent factor processes. - Explore Bayesian inference and computation for studying dynamic factor structure.
Company Stock in Pension Funds.	William E. Even, D. Macpherson (2004)	- Examine issues surrounding pension funds investing in their own company's stock. - Review benefits and costs of investing in company stock. - Review legislative environment surrounding company stock holdings.
Continuous-Time Mean–Variance Optimization for Defined Contribution Pension Funds with Regime-Switching.	Zhiping Chen, Liyuan Wang, Ping Chen, H. Yao (2019)	- Investigate a continuous-time defined contribution pension fund investment problem using the mean-variance criterion. - Construct a framework under a Markovian regime-switching market. - Conduct numerical analyses to illustrate the properties of the optimal strategy and the effects of model parameters.
Defined contribution pension planning with a stochastic interest rate and mean-reverting returns under the hyperbolic absolute risk aversion preference.	Hao Chang, Chunfeng Wang, Zhenming Fang, Dan Ma (2019)	- To model stochastic interest rates and market prices of risk. - To incorporate different risk preferences using hyperbolic absolute risk aversion utility. - To find optimal investment strategies using Legendre transform-dual theory.
Determinant Factors of The Investment Performance of Voluntary Pension Funds in Romania.	Victoria Şeulean, L. Mos (2010)	- To analyze the factors that determine changes in the value of the fund unit. - To establish the relationship between total asset allocation and the value of the fund unit.
Dynamic discrete-time portfolio selection for defined contribution pension funds with inflation risk.	H. Yao, Ping Chen, Miao Zhang, Xun Li (2020)	- Investigate a multi-period asset allocation problem for a defined contribution pension fund facing stochastic inflation. - Address the problem in a discrete-time setting. - Illustrate the impacts of inflation, investment horizon, estimation error, and superannuation guarantee rate on the efficient frontier.
Effect of Extra Contribution on Stochastic Optimal Investment Strategies for DC Pension with Stochastic Salary under the Affine Interest Rate Model.	K. Njoku, B. Osu, E. Akpanibah, Rosemary N. Ujumadu (2017)	- To study optimal investment strategies for defined contribution (DC) Pension with extra contribution. - To develop a mathematical framework using the Hamilton-Jacobi-Bellman equation and Dual theory for these strategies.
Effect of Inflation on Stochastic Optimal Investment Strategies for DC Pension under the Affine Interest Rate Model.	B. Osu, K. Njoku (2019)	- Investigate the effect of inflation on optimal investment strategies for DC Pension. - Assess the extent of damage inflation causes to propose an optimum amortization fund. - Analyze the effects of inflation parameters and CRRA utility function on investment strategies.
Equilibrium Investment Strategy for a DC Plan with Partial Information and Mean–Variance Criterion.	Yongwu Li, Shouyang Wang, Yan Zeng, H. Qiao (2017)	- Study a mean–variance portfolio selection problem with partial information for a defined-contribution (DC) pension scheme. - Derive an equilibrium investment strategy by solving an extended Hamilton–Jacobi–Bellman (HJB) system of equations.
Equilibrium investment strategy for defined-contribution pension schemes with generalized mean–variance criterion and mortality risk.	Huiling Wu, Yan Zeng (2015)	- Investigate the equilibrium investment strategy for defined-contribution pension schemes with a generalized mean–variance criterion and mortality risk. - Derive the optimal investment strategy for a defined-contribution pension scheme under the generalized mean–variance criterion and mortality risk.
Estimation of Stochastic Volatility Models.	F. Bartolucci, G. Luca (2002)	- Consider the stochastic volatility model and its estimation problems. - Overcome difficulties in evaluating and maximizing likelihood using a quadrature method. - Apply the technique to stock exchange indexes. - Compare competitor ARCH-type models with quadrature method.

Evaluating the Financial Performance of Pension Funds.	R. Hinz, H. Rudolph, P. Antolín, Juan Yermo (2010)	- Summarize the studies conducted and their main findings. - Develop approaches to evaluate performance of pension funds. - Interpret and discuss the findings with expert commentary.
Impact of Unfunded Pension Obligations on Credit Quality of State Governments.	Christine R. Martell, Sharon N. Kioko, Tima T. Moldogaziev (2013)	- Review the funding status of state-administered pension plans and their impact on state credit quality. - Serve as a benchmark to assess the impact of new reporting requirements on the market's response to unfunded pension liabilities.
Labor Market Uncertainty and Pension System Performance.	O. Mitchell, J. Turner (2009)	- Review existing studies on human capital risk and pension outcomes. - Offer a framework to understand how human capital risk influences pension outcomes. - Provide guidance for future analysts on assessing sensitivity of pension plans to labor income uncertainty.
Legendre Transform-Dual Solution for a Class of Investment and Consumption Problems with HARA Utility.	Hao Chang, X. Rong (2014)	- To provide a Legendre transform method for investment and consumption problems. - To obtain the optimal investment and consumption strategy for HARA utility using dynamic programming and Legendre transform.
Life-Cycle Finance and the Design of Pension Plans.	Bodie, Zvi, Detemple, Jérôme, Rindisbacher, Marcel (2009)	- Review recent scientific literature on consumer financial decisions over the life cycle. - Summarize empirical literature on household behavior regarding saving, investing, and insuring consumption in old age. - Comment on practical implications for pension system design and outline future research areas.
Life-Cycle Funds.	Luis M. Viceira (2007)	- Review recent advances in academic models of asset allocation for long-term investors. - Propose improvements to life-cycle funds, including heterogeneity in risk tolerance and specificity to defined-contribution plans. - Provide guidance for choosing target retirement dates based on life-expectancy.
Linking agent-based models and stochastic models of financial markets.	Ling Feng, Baowen Li, B. Podobnik, Tobias Preis, H. Stanley (2012)	- Construct an agent-based model to demonstrate how fat-tailed distributions in financial returns are influenced by traders using similar technical strategies. - Extend the agent-based model to a stochastic model to derive scaling relations for long-term memory in financial markets. - Provide a behavioral explanation for stochastic models in financial systems and a method to parameterize these models using market data.
Measuring and modeling variation in the risk-return tradeoff.	M. Lettau, S. Ludvigson (2005)	- Explore if stock return predictability can be explained by changes in stock market volatility. - Analyze how the mean return per unit risk changes over time. - Review existing knowledge and provide new empirical evidence on the time-series evolution of the risk-return tradeoff for stock market investment.
On the Effect of Inflation and Impact of Hedging on Pension Wealth Generation Strategies under the Geometric Brownian Motion Model.	B. Osu, K. Njoku, B. Oruh (2019)	- Investigate the effect of inflation on optimal investment strategies in a DC pension scheme. - Analyze the impact of hedging using inflation-indexed bonds and inflation-linked stocks. - Introduce an optimum amortization fund to dampen inflation's effects.
On the Effect of Stochastic Extra Contribution on Optimal Investment Strategies for Stochastic Salary Under the Affine Interest Rate Model in a DC pension Fund.	B. Osu, E. Akpanibah, C. NjokuK.N. (2017)	- To study the optimal investment strategy for a plan contributor in a defined contribution (DC) pension scheme with stochastic salary and stochastic extra contribution under the affine interest rate model. - To analyze the effect of the extra contribution rate on the optimal investment strategies in various investments.
Optimal asset allocation in a stochastic factor model-an overview and open problems.	T. Zariphopoulou (2009)	- Provide an overview of the optimal investment problem in a market with a correlated stochastic factor. - Evaluate the performance of investment strategies using two criteria: expected utility from terminal wealth and forward investment performance approach.
Optimal Consumption and Portfolio Decision with Convertible Bond in Affine Interest Rate and Heston's SV Framework.	Hao Chang, Xueyan Li (2016)	- Solve an optimal investment-consumption problem with stochastic affine interest rate and stochastic volatility. - Derive the HJB equation and explicit expressions of optimal investment-consumption strategies.
Optimal DC pension investment with square-root factor processes under stochastic income and inflation risks.	Yumo Zhang (2022)	- Investigate optimal defined contribution (DC) pension investment problems under stochastic income and inflation risks. - Use a backward stochastic differential equation (BSDE) approach to tackle non-Markovian structures. - Provide closed-form expressions for optimal investment strategies and value functions for power, logarithmic, and exponential utility functions.
Optimal Design and Regulation of Funded Pension Schemes.	A. Bovenberg, R. Mehlkopf (2013)	- Review the literature on the optimal design and regulation of funded pension schemes. - Characterize optimal saving and investment over an individual's life cycle. - Explore various extensions of the model (e.g., additional financial risk factors, stochastic human capital, individual preferences).

Optimal investment and benefit payment strategies for TB pension plans with stochastic interest rate under the HARA utility.	Yijun Wang, Huanying Zhang, Zilan Liu, Ya Huang (2023)	- To incorporate realistic financial market conditions using the Vasicek model and HARA utility. - To construct the Hamilton–Jacobi–Bellman (HJB) equation and obtain explicit expressions for optimal strategies.
Optimal Investment of DC Pension Plan Under Short-Selling Constraints and Portfolio Insurance.	Yinghui Dong, Harry Zheng (2018)	- Maximize the expected S-shaped utility of the terminal wealth exceeding a minimum guarantee. - Apply the dual control method to solve the problem. - Show the impact of S-shaped utility, short-selling constraints, and portfolio insurance on the optimal terminal wealth.
Optimal Investment Strategies for DC Pension with Stochastic Salary under the Affine Interest Rate Model.	Chu-bing Zhang, X. Rong (2013)	To study the optimal investment strategies of DC pension with stochastic interest rate and stochastic salary.
Optimal investment strategies for the HARA utility under the constant elasticity of variance model.	E. Jung, J. Kim (2012)	To find the optimal investment strategies for the HARA utility under the constant elasticity of variance model.
Optimal investment strategy for a DC pension plan with mispricing under the Heston model.	Jie Ma, Hui Zhao, X. Rong (2020)	- To address the optimal investment problem for a defined contribution (DC) pension plan with mispricing. - To maximize the expected utility for the power utility function of terminal wealth.
Optimal Investment Strategy under the CEV Model with Stochastic Interest Rate.	Yong He, Peimin Chen (2020)	- Study an optimal investment strategy problem using a CEV process and stochastic interest rate. - Maximize the expected utility of terminal wealth.
Optimal portfolios for the DC pension fund with mispricing under the HARA utility framework.	Zilan Liu, Yijun Wang, Ya Huang, Jieming Zhou (2022)	- To adopt the HARA utility framework to describe risk performance. - To construct the HJB equation and obtain explicit expressions for optimal portfolio choices. - To conduct numerical analysis to illustrate sensitivity to market and contribution parameters.
Pension Fund Asset Allocation: A Mean-Variance Model with CVaR Constraints.	Yibing Chen, Xiaolei Sun, Jianping Li (2017)	- Review important aspects of asset allocation for large Social Security Reserve Funds. - Present the mean-variance model with CVaR constraints as an asset allocation methodology.
Pension Fund Investments: Stocks or Bonds?	Frank de Jong (2003)	- Review the investment policy of collective pension plans. - Explore reasons for investing in equities. - Assess the value and optimality of conditional indexation rules.
Pension Fund Management in a Stochastic Optimization Framework.	S. Vitali (2018)	- Analyze problems regarding pension funds. - Find optimal solutions through stochastic optimization. - Optimize risk-reward profiles for sponsors. - Suggest best allocation for individuals to achieve target pension benefits.
Pension Fund Mathematics.	A. Cairns (2006)	- Review the role of mathematics in running pension funds. - Consider separately defined benefit (DB) and defined contribution (DC) pension funds.
Pension Fund Risk Management: Financial and Actuarial Modeling.	Marco Micocci, Greg N. Gregoriou, and Giovanni Batista Masala (2010)	- Develop a comprehensive framework for pension fund risk management. - Provide a detailed analysis of financial and actuarial modeling techniques. - Evaluate the effectiveness of these models in managing risk and ensuring sustainability.
Pension funds and Market Efficiency: A review.	Ashok Thomas, Luca Spataro (2013)	- Provide an updated review of theoretical and empirical advances in the relationship between pension funds and market efficiency.
Portfolio Choice with Stochastic Investment Opportunities: a User's Guide.	Ren Liu, Johannes Muhle-Karbe (2013)	- Provide methods for computing candidate optimal portfolios using stochastic control. - Verify the optimality of portfolios using convex duality. - Emphasize long-horizon asymptotics for tractable results.
Portfolio Optimization for Pension Purposes: Literature Review.	Moreira, Leonardo, Santos, Igor Leão Dos, Gonzalez, Pedro Henrique (2025)	- Identify persistent challenges and gaps in the literature on pension portfolio optimization models. - Investigate barriers to the effective implementation of these models.
Portfolio Performance Evaluation.	G. Aragon, W. Ferson (2007)	- Review methods for measuring portfolio performance. - Examine selectivity and market timing ability using newer performance measures.
Portfolio Selection with Liability and Affine Interest Rate in the HARA Utility Framework.	Hao Chang, Kai Chang, Ji-mei Lu (2014)	- To study an asset and liability management problem with stochastic interest rates and liability processes. - To introduce liability processes into a continuous-time dynamic portfolio selection problem with affine interest rate models. - To investigate the optimal investment strategy for an asset and liability management problem within the HARA utility framework.
Protecting and Politicizing Public Pension Fund Assets: Empirical Evidence on the Effects of Governance Structures and Practices.	David Hess (2005)	- Review the impact of political influence on investment performance. - Expand on existing research by considering additional governance factors. - Inform policy decisions regarding member-elected trustees and trustee training.

Public Pension Fund Management: Best Practice and International Experience.	Rozanov, Andrew (2015)	- Review the core principles and best practices of public pension fund management, focusing on governance and investment philosophy. - Compare and contrast three distinct models of institutional fund management: the Norway model, the Yale/Australia Future Fund model, and the Canada model.
Public Pension Plan Asset Allocations.	Youngkyun Park (2009)	- Examine the effect of investment performance on employer contribution volatility. - Examine the volatility in employer contribution rates caused by higher-return-seeking/higher-risk investment portfolios. - Investigate whether plan sponsors will increase fixed-income investments to reduce volatility.
Recent Trends in Canadian Defined-Benefit Pension Sector Investment and Risk Management.	Eric Tuer, Elizabeth Woodman (2005)	- Examine how funding deficits, focus on plan liabilities, low yield environment, and changing investment beliefs influence investment decisions in the Canadian DB pension sector. - Review the funding of DB plans, changing views on the equity-risk premium, and the shift towards liability-centered approaches to investment.
Robust optimal strategies of DC pension plans with stochastic volatility and stochastic income under mean-variance criteria.	Hao Chang, Jiaao Li, Hui Zhao (2021)	- To study a robust optimal investment problem under the mean-variance criterion for a DC pension plan with an ambiguity-averse member. - To establish a continuous-time mean-variance model with ambiguity aversion for a DC pension plan. - To derive closed-form expressions for robust efficient strategy and efficient frontier.
Saving and Investing Over the Life Cycle and the Role of Collective Pension Funds.	Bovenberg, Lans, Koijen, Ralph, Nijman, Theo, Teulings, Coen (2007)	- Investigate constraints on individual decision-making and the role of collective pension schemes. - Explore extensions to the benchmark model, including more complex human capital models and additional risk factors. - Evaluate the costs and benefits of collective versus individual pension schemes.
Sovereign Wealth Fund Asset Allocations—Some Stylized Facts on the Norway Pension Fund Global.	Michael G. Papaioannou, Bayasgalan Rentsendorj (2015)	- To analyze the strategic asset allocation of the Norway Government Pension Fund Global (GPFG) in relation to the Markowitz model. - To compare GPFG's asset allocation results with those of other SWFs to determine broader conformity with the Markowitz model.
Stochastic Volatility.	T. Andersen, Luca Benzoni (2018)	- Provide an overview of stochastic volatility models. - Illustrate applications to practical finance problems. - Highlight utility in pricing and hedging applications. - Address challenges in estimation and inference.
Stochastic Volatility: Modeling and Asymptotic Approaches to Option Pricing and Portfolio Selection.	Matthew J. Lorig, R. Sircar (2016)	- Review how stochastic volatility can be modeled. - Quantify how stochastic volatility affects option prices. - Quantify how stochastic volatility affects investment strategies.
Stochastic Volatility: Origins and Overview.	N. Shephard, T. Andersen (2008)	- Review the development of stochastic volatility in a modeling and historical context. - Highlight recent trends in the literature. - Provide a selective overview of the SV literature.
Strategic Asset Allocation: Portfolio Choice for Long-Term Investors.	Christian Schlag (2003)	- To investigate long-term bonds, stock market investments, and continuous-time models. - To discuss the impact of labor income shocks on portfolio choice. - To investigate investment over the life cycle.
Technical Paper Series Congressional Budget Office Washington, DC.	Daniel M. Chin, John Geweke (2000)	- To explore correlations between equity returns and market fundamentals. - To evaluate random returns models versus models based on market fundamentals for simulating future equity returns. - To quantify the trade-off between risk and return in policy simulations.
The Effects of Pension Funds on Markets Performance: A Review.	Ashok Thomas, Luca Spataro (2016)	Uncover stylized facts to inspire theoretical models and inform policy debates.
The Impact of Stocks and Bonds on Pension Fund Performance.	Božena Chovancová, Jaroslav Hudcovský, Anna Kotásková (2019)	- Investigate the connection between stock and bond markets and pension funds. - Confront the relationship between the pension market and representative stock and bond market indexes.
The Legendre transform-dual-asymptotic solution for optimal investment strategy with random incomes.	Jinyang Liu, Sheng Li, Yongze He, Boping Tian, Li Deng (2023)	- To use a constant variance elasticity (CEV) model for the price of the risky asset. - To consider any correlation coefficient between the income risk and the risk of risky asset. - To apply the Legendre transformation, dual theory, and asymptotic expansion approach to obtain an asymptotic strategy for the exponential utility function.
Unlocking the pension fund performance: A systematic literature review approach.	Bogdan, Siniša, Brmalj, Natali, Olgić Draženović, Bojana (2025)	- Identify the determinants of pension fund performance that contribute to competitiveness and governance. - Conduct a systematic literature review and bibliometric analysis to understand these determinants. - Examine the relationship between the quality of a country's pension system and the origin of relevant research.

Source: authors' elaboration

In general, there were several numerical methods detected in the papers reviewed. Some are related exclusively to the pensions area and others to the finance areas. A summary of these methods is as follows (in alphabetical order): Agent-based model, ARCH-type models with quadrature method, Backward stochastic differential equation (BSDE), Bayesian inference and computation, Constant variance elasticity (CEV) model, Continuous-time mean-variance model, Convex duality, CRRA utility function, Generalized mean–variance, Hamilton–Jacobi–Bellman (HJB) equation, HARA utility framework, Heston's SV Framework, Legendre transformation, Markowitz model, Mean-variance model with CVaR constraints, Monte Carlo simulations, Nonlinear second-order partial differential equations, Stochastic optimization, Time-series. The pensions area topics in which these numerical methods are applied are as follows (in alphabetical order): Asset allocation, Expected utility, Funding of DB plans, Investment over the life cycle, Investment performance, Life-cycle funds, Optimal saving, Performance of pension funds, Trade-off between risk and return.

To report on the relevant methods for each source of evidence relating to the question and objectives of this work, see table 2. The methods are grouped by analyzing the methodologies in each of the papers.

Table 2.
Methods mentioned in the total of 64 peer-reviewed papers.

Authors	Method
F. Bartolucci, G. Luca (2002)	ARCH-type model
Yumo Zhang (2022)	Backward stochastic differential equation (BSDE)
Yong He, Peimin Chen (2020)	CEV process
B. Osu, K. Njoku (2019)	CRR utility
Bogentoft, Erik, Edwin Romeijn, H., Uryasev, Stanislav (2001); Yibing Chen, Xiaolei Sun, Jianping Li (2017)	CVaR model
Hao Chang, X. Rong (2013)	Expected discounted utility
K. Njoku, B. Osu, E. Akpanibah, Rosemary N. Ujumadu (2017); Hao Chang, Xueyan Li (2016); Yongwu Li, Shouyang Wang, Yan Zeng, H. Qiao (2017)	Hamilton-Jacobi-Bellman equation
Yijun Wang, Huanying Zhang, Zilan Liu, Ya Huang (2023); E. Jung, J. Kim (2012); Hao Chang, X. Rong (2014); Hao Chang, Kai Chang, Ji-mei Lu (2014); Zilan Liu, Yijun Wang, Ya Huang, Jieming Zhou (2022)	HARA utility model
B. Osu, K. Njoku, B. Oruh (2019)	Inflation-indexed bonds
Hao Chang, Chunfeng Wang, Zhenming Fang, Dan Ma (2019); Jinyang Liu, Sheng Li, Yongze He, Boping Tian, Li Deng (2023)	Legendre transform
Bovenberg, A. Lans, Mehlkopf, Roel (2013); Bodie, Zvi, Detemple, Jérôme, Rindisbacher, Marcel (2009); Bovenberg, Lans, Koijen, Ralph, Nijman, Theo, Teulings, Coen (2007); Luis M. Viceira (2007); Lans Bovenberg, R. Mehlkopf (2014)	Life-cycle
Moreira, Leonardo, Santos, Igor Leão Dos, Gonzalez, Pedro Henrique (2025); R. Hinz, H. Rudolph, P. Antolín, Juan Yermo (2010); Christine R. Martell, Sharon N. Kioko, Tima T. Moldogaziev (2013); O. Mitchell, J. Turner (2009); A. Bovenberg, R. Mehlkopf (2013); S. Vitali (2018); A. Cairns (2006); Ashok Thomas, Luca Spataro (2013); G. Aragon, W. Ferson (2007); Eric Tuer, Elizabeth Woodman (2005); N. Shephard, T. Andersen (2008); Ashok Thomas, Luca Spataro (2016)	Literature/Systematic review
Zhiping Chen, Liyuan Wang, Ping Chen, H. Yao (2019); Michael G. Papaioannou, Bayasgalan Rentsendorj (2015)	Markov model

Huiling Wu, Yan Zeng (2015); H. Yao, Yongzeng Lai, Q. Ma, Minjie Jian (2014); Hao Chang, Jiaao Li, Hui Zhao (2021)	Mean-Variance model
David Tuesta, Carlos Herrera (2014)	Monte Carlo
Bogdan, Siniša, Brmalj, Natali, Olgić Draženović, Bojana (2025); H. Yao, Ping Chen, Miao Zhang, Xun Li (2020); Xiaoyi Tang, Wei Liu, Wanyin Wu, Yijun Hu (2024); Jie Ma, Hui Zhao, X. Rong (2020); Giuseppina Cannas (2011); William E. Even, D. Macpherson (2004); Frank de Jong (2003); Victoria Şeulean, L. Mos (2010); Christian Schlag (2003); Božena Chovancová, Jaroslav Hudcovský, Anna Kotásková (2019)	Optimal asset allocation
Marco Micocci, Greg N. Gregoriou, and Giovanni Batista Masala (2010)	Pension Fund Risk Management
David Hess (2005)	Political influence on investment performance
Rozanov, Andrew (2015); Youngkyun Park (2009); Daniel M. Chin, John Geweke (2000)	Public Pension Fund Management
Yinghui Dong, Harry Zheng (2018)	S-shaped utility
Chu-bing Zhang, X. Rong (2013); B. Osu, E. Akpanibah, C. NjokuK.N. (2017); Ling Feng, Baowen Li, B. Podobnik, Tobias Preis, H. Stanley (2012); M. Lettau, S. Ludvigson (2005); T. Zariphopoulou (2009); Ren Liu, Johannes Muhle-Karbe (2013); T. Andersen, Luca Benzoni (2008); Matthew J. Lorig, R. Sircar (2016)	Stochastic methods (in general)
O. Aguilar, M. West (2000)	Time Series

Source: Author elaboration with resources of OpenAI (2025b).

Table 2 shows that there are several numerical methods explored. Some are specific methods, and others are stochastic methods in general. It is also found that some papers refer to systematic reviews or literature exploration of the topic in the pension area, mainly. By making use of the AI Mapify, table 3 shows a synthesis of the issues detected at the papers and organized as a conceptual map.

Table 3.
Conceptual map derived from the main topic "optimization, managing and performance of pension funds".

Optimization, managing and performance of pension funds				
Trends, challenges and perspectives in pension portfolio optimization.	Stochastic modeling and volatility in pension fund management.	Determining factors and performance of pension funds.	Quantitative models and mathematics for pension fund management.	Investment strategies and asset allocation in pension funds.

Source: Author's elaboration based on resources from OpenAI (2025c).

At the top in table 3, the general topic is presented "optimization, managing and performance of pension funds" followed by 5 other topics detected derived from the main one. Each of these 5 areas has its own relevant subareas that are shown in table 4.

Table 4.
Subareas derived from the previous topics in table 3.

Source: Author's elaboration with sources of OpenAI (2025c).

Discussion

In this scoping review, a total of 64 papers were identified that addressed numerical methods in the asset management of investment portfolios. The question defined in this work specifically referred to methods that ensure financial viability and sustainability. However, the methods found in the literature that met the eligibility criteria very rarely refer to that objective specifically. To summarize, the eligibility criteria were as follows: published in English or Spanish between 2000 and 2025; a PDF was available, and only in the pensions or finance areas.

Our findings are that the numerical methods found are related mainly to a specific stochastic method, and some have an actual application to databases. In other cases, the methods are only mentioned without giving a numerical example. In either case, the results are that only a few mention that the applications were made to explore if these ensure financial viability and sustainability. Moreover, the studies included clearly support that stochastic methods perform better than deterministic ones, mentioning that market volatility needs to be modelled with robust numerical methods.

Our scoping review has some limitations. It was only possible to include a sample of rapid reviews from websites where the PDF was available; this leaves behind the peer-reviewed papers that require a fee to visualize the full-text. Also, the period was limited to the past 25 years, when it is known that the numerical methods date back several more years. Another limitation of this work is that, as of today, there are many AI applications available on the internet that can produce many more comparisons and conceptual maps than the 3 that were selected in this work.

Finally, it is important to note that a scoping review aims to identify gaps in the literature which may give recommendations for future scoping and systematic reviews. The results found are that there are several papers related to the topic defined in this study: *numerical simulations in the assets administration of pension funds*. However, to ensure financial viability and sustainability, only a few determine this objective. There is a gap in the literature regarding the numerical application of methods explicitly mentioned in the pensions area.

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