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# Creative industries innovation using galois group theory

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## ABSTRACT

To address creative industries' challenges with an innovative Fuzzy Logic approach. A robust methodological structure using Galois Group Theory and an intuitive application for decision making under uncertain conditions is proposed. Results conclude that products with different characteristics, properties and peculiarities can be grouped with a high confidence level through an intuitive fuzzy methodological approach. The present study pretends to shed light in grouping methodologies, attending challenges in which traditional grouping methods, which are mainly driven by trial and error efforts have not succeed before. The methodology is applied in order to group a specific city's tourism products; the attempt is to achieve an effective decision making process. The originality of the study relies on the capacity and flexibility of the model to analyze different characteristics of diverse products under subjective and uncertain conditions and the implementation of solid theories from a fuzzy logic standpoint.

**Keywords:** Uncertainty, Fuzzy logic, Group, Fuzzy Sets.

## RESUMEN

Atender los desafíos de las industrias creativas con un enfoque innovador a partir de herramientas de la Lógica Difusa. Se propone una estructura metodológica sólida usando la Teoría de Grupos de Galois y una aplicación intuitiva para la toma de decisiones en condiciones de incertidumbre. Resultados concluyen que productos con diferentes características, propiedades y peculiaridades se pueden agrupar con un alto nivel de certeza. La metodología se aplica con el fin de agrupar los productos turísticos de una ciudad específica; el intento es lograr un proceso de toma de decisiones eficaz. La originalidad del estudio se basa en la capacidad y la flexibilidad del modelo para analizar las diferentes características de los diversos productos en condiciones subjetivas e inciertas y la aplicación de teorías sólidas desde el punto de vista de lógica difusa.

**Palabras clave:** Incertidumbre, Lógica Difusa, Grupos, Sets Difusos.

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# INTRODUCTION

***Creative Cities Supporting Creative Industries***

The concept of the Creative Cities was initially introduced in the earliest 1990's, by the author Charles Landry. His initial concept appeared from the need of an urban, economic, social and cultural transformation, which had to direct the roll that the cities could play around the dramatic changes that appeared in the new global environment. The cities therefore had to be transformed in wealth creation hubs (Landry, 2000) under the new global dynamics that were developing.

Florida (2012) describes the concept of a new social class defined as the "Creative Class", individuals whose talent and potential transformed the way goods and services were produced, employments and enterprises were created, and in general terms; wealth is created. The original idea resulted from the analysis of elements that detonated regional development; a key aspect is the retention and attraction of enterprises which main actions include the input of human talent to their companies. In practical terms the creative class detonates the appearance of enterprises, which by their activities generate wealth in the region.

The origin of the "creative city" can be approached and understood by these two key concepts and their authors. Although in words of Landry we find an intrinsically more strategic approach to be applied by the city planners, and in words of Florida we find a more economic and social component in which the triggering factor for regional success is the human talent (Florida 2012), in the association and combination of their concepts we can establish common variables that in terms of this study, will be considered to develop the thesis and value added proposals.

A wide range of definitions of "creative industries" have been developed in recent years; it appears to be evolving as our understanding on the positive impacts that these kinds of industries bring to the economy. As a first approach to the evolution of the definition Hall (2000) points out, all the transitions starting from the manufacturing economy, to the information economy up to the new cultural economy (where the concept of cultural industry is born) led the path to an economic restructure and regeneration. In the present paper we will follow the definition of the UNCTAD (2010), which establishes: "any economic activity producing symbolic products with a heavy reliance on intellectual property and for as wide a market as possible". The same institution catalogued the creative industries in 4 major categories: Heritage: identified as the origin of all forms of arts and the soul of cultural and creative industries. It brings together cultural aspects from the historical, anthropological, ethnic, aesthetic and societal viewpoints, influences creativity and is the origin of a number of heritage goods and services as well as cultural activities. Arts: This group includes creative industries based purely on art and culture. Artwork is inspired by heritage, identity values and symbolic meaning. Media: This group covers two subgroups of media that produce creative content with the purpose of communicating with large audiences ("new media" is classified separately). Functional creations: This group comprises more demand-driven and services-oriented industries creating goods and services with functional purposes.

Creative cities must open opportunities in diverse areas for the creation and development of creative industries. Creative Metropoles (2010) propose one key study in the development of strategies towards the enhancement of creative industries. This study is financed by the European Union and it establishes the relations and results that different cities in the EU have had applying processes of creativity. The main objective of this study is to communicate the different experiences and real examples of the cities towards in-vestment in creative industries and creative cities theories. The examples found in the study demonstrate how cities like: Barcelona, Oslo, Birmingham, Riga, Stockholm, Tallinn, Amsterdam, Helsinki, Berlin y Warsaw had experienced through the impulse that inputs of creative cities theories have had.

***Innovation Towards Creative Industries***

In our days there is no manager or decision maker that could affirm that innovation does not carry competitiveness, it is in some way a given fact. Porter (1990) stated, "A nation's competitiveness depends on the capacity of its industry to innovate and upgrade. Companies gain advantage against the world's best competitors because of generating innovations".

The Oslo Manual (2005) indicates that innovation can be characterized into four kinds: product innovation, process innovation, organizational innovations and marketing innovation: Product innovation implies significant changes of the characteristics in products or services. They can include completely new products or services and the significant improvement of existing ones. Process innovation refers to significant changes in production and distribution methods. Organizational innovations refer to the establishment of new methods of organization. These can be changes in the practices of the enterprises, in the organization of the workplace or in the external relations of the firm. Marketing innovation implies the establishment of new commercialization methods. These can include changes in the design and envelope of products, in the promotion and colocation of goods and in the methods of pricing in products or services.

In the present work we propose a methodology to aid decision makers in marketing innovation process, by offering a specific grouping technique of products which has showed relevant results in practice. In recent years more attention has been attracted to this concept due to the fact that "the development of new marketing tools and methods plays an important role in the evolution of industries." Chen (2004).

Both innovation and creativity present highly uncertain bases, as for the endogenous and exogenous elements that surround them; therefore the adoption of decisions under a fuzzy approach has gained special relevance. Studies with a fuzzy-oriented standpoint have been increasing since the last century and have proven efficacy while dealing with complex phenomena.

***Fuzzy Logic***

It is widely accepted that decision-making process involves uncertainty, imprecision and imperfect or vague information. As stated by Bellman & Zadeh (1970) "much of the decision making in the real world takes place in an environment in which the goals, the constraints and the consequences of possible actions are not known precisely". The theory of decision under uncertainty initializes with the appearance of the article Fuzzy sets. Information and Control, Zadeh (1965), and has proven efficiency handling incomplete and uncertain knowledge information see Ribeiro (1996). The theory of Fuzzy Sets has been applied in the field of the formal sciences; nonetheless in the past 44 years researchers from all over the world have been publishing diverse research studies with applications in varied fields of knowledge.

The relation that is established between the products to evaluate and the variables that characterize them as criteria for aggrupation and creation of synergies starts from the proposal of Kaufmann & Gil-Aluja (1998). The purpose of this work is to classify and group, different products that could by creation of synergies, increase their attraction as a whole. The method to classify and group these products will have as a foundation Galois' group theory; see Keropyan & Gil-Lafuente (2013) and the theory of fuzzy sets, see (Gil-Aluja, et al., 2011). These approximations admit us to construct a generalized model adapted to the conditions of expectancy and uncertainty.

## PRELIMINARIES

The origins of this study rely on the importance and relevance that emerging economies thus, emergent cities are exhibiting. Moreover the positive impact in which innovation management under a fuzzy approach could generate, finding in an efficient way existing connections, relations, and similarities between products; creating synergies and raising the level of attraction and competitiveness of cities. These efforts in organization and synergy should exert benefits in the economic, social and environmental realm by producing greater effects than the sum of the individual labors.

### *The model*

The model that we build aims to modernize the methods used before in the field of municipality's touristic management. The model is a different and structurally improved way of establishing groups to create synergies. The optimal grouping can lead to join the most affine products in order to share capacity resources and in general terms help decision makers to create better strategies in order to increase the allurement, appeal and attraction of a city.

In our model, we make a transition from verbal semantics to the corresponding numerical semantics in order to be able to group the most affine city's highlights, matching them by the valuation of their inherent characteristics, qualities and peculiarities. The model allows flexible procurement of information by empowering city experts and decision makers in the valuation of the touristic place's characteristics and their desired similarity level.

The adequacy of the model is very important in terms of measuring well the characteristics of the city's highlights and determining if these characteristics can be match with other highlights characteristics to build strong synergies. The characteristics are not always objective. The model we propose lets us introduce subjective information for certain special cases where measurement is possible. Although there may exist some objective characteristics we have to accept the fact that the transition from verbal semantics to numerical semantics is subjective for those special cases that could have been measured, Gil-Lafuente (2002).

In general, the adoption and further application of Galois group theory has multiple significances:

i) At first, it allows to establish different levels of synergies that could be created as of the inherent characteristics of the tourist attractions analyzed.

ii) Secondary, once the level of synergies has been established, the model allows knowing precisely, which are the specific characteristics that foster the optimization of the groups.

iii) Thirdly, the model permits the selection on which of the characteristics the decision maker wants to prioritize in a specific environment and strategic requirements.

Galois group theory has been proven efficient in different fields since the "order- or structure-preserving passage between two worlds of our imagination - and thus are inherent in human thinking wherever logical or mathematical reasoning about certain hierarchical structures is involved" Denecke et al., (2004).

Other applications that have conducted with success the application of Galois Lattices can be found in the aggrupation of stakeholders for a better administration of enterprises, see Gil-Lafuente & Paula (2013), and in human resources areas, with a personnel selection model Keropyan & Gil-Lafuente (2013).

***Studied City***

In order to understand the application of the method, we will briefly describe the profile of the city that has been chosen to develop the new touristic grouping model. As the reader will notice some of the main reasons of choosing this specific metropolis are because of the great importance that tourism has on its economy, the strategic location of the municipality and the need of fostering new management methods in order to maintain the attraction and appeal of visitors.

Morelia is a city located in the center of the Mexican Republic; it is the capital of the state of Michoacán de Ocampo. It is immersed in the Mega – region called "Greater Mexico City", where approximately 45 million people live and generates $290,000 million in LRP, more than half of the whole nation, Florida (2008).

Geographically Morelia finds itself 303 km from the capital of México City. Approximately 295 km to the north we find the city of Guadalajara, Jalisco, city known because of the culture, industry and the attractiveness to diverse businesses. About 196 km from Morelia, we find the city of Querétaro, recognized because of all the industrial activities that are held out. One of the most important ports of the country, Lázaro Cárdenas port, finds itself around 387 km from the capital of the State. The city also connects to different metropolises of México by its wide railroad infrastructure and the international Airport "General Francisco Mújica".

In economic aspects, the city has an overall gross domestic product of 7,774.5 dollars per capita, when the mean in the republic is 9,980 dollars. The city raised its gross domestic product from the 2003 to 2008 by 15%. The main economic activities of the city are tourism, education and commerce. The city reaches 1,606,399 economically active citizens which 1,554,720 are employed.

In terms of tourism, the city of Morelia is one of the first touristic destinations in the country due to its architectonical, cultural and historic legacy. The city also connects with a series of natural destinations, which increase the affluence of tourists. The city has over 110 lodging establishments and only in 2010 Morelia attracted 2,449,805 national tourists and 269,179 international tourists.
Referring to amenities, the city has a wide variety of theaters, museums, cinemas, bars and entertaining establishments, which nurture the popular culture and generate great attractiveness for the creation of micro and small enterprises.

In terms of education, the city of Morelia offers 882 educational facilities, 7,744 classrooms, 81 libraries, 103 workshops and 165 laboratories. Is in this city where one of the most important Universities of Mexico is established, the “Universidad Michoacana de San Nicolas de Hidalgo” where diverse ambits of science are studied such as health, administration and accountancy, legal, exact sciences, humanities, engineering and architecture. There is also a vast offer of postgraduate studies. In this university around 32,000 students are active. An Institute of Technology resides in the city; around 4,650 students specialize in technological fields of the knowledge. In general terms there are 9 public institutes of advance studies and 15 private ones.

In terms of culture, the city of Morelia is a national exponent having some of the most important artistic, musical and cinema events, as well as diverse expositions. Morelia has a total population of 729,279 inhabitants, 94.2% of them are alphabets and the standards of human development are in the top ranked of the country.  In the city we can find the “Conservatorio Musical” musical school founded in 1743, which provides the city of culture and artists in different specializations. In other cultural aspects, Morelia is named human heritage in 1991 by the UNESCO, association that also gave the city the title of “Sanctuary of the Monarch Butterfly”, and sponsor of the “Day of the Dead National Celebration City”, “La Pirekua Musical Heritage of the Humanity”, and “Traditional Mexican cuisine - ancestral, ongoing community culture, the Michoacán paradigm”.

If in general terms the city has a great cultural reservoir, great affluence of tourism, and attractive popular amenities, the development of the industrial tissue is incipient, this affects in terms of providing rewarding employments to the citizens of the city. In terms of industry, the city has over 16 mining economical units, where around 100 people work, 3143 manufacturing enterprises where 14,606 people work. In 2008 only 16 licenses for industrial land use were petitioned. The city has an industrial park where 180 enterprises offer around 9,000 employments, most of the enterprises established have only distributing activity and the manufacturing enterprises are small or medium size companies. All the data was retrieved from the National Institute of Statistics and Geography (INEGI).

## APPLICATION

When conducting traditional grouping of elements, trial and error methods are employed, therefore the confidence level in which the inherent characteristics of the studied products relate to each other tends to be scarce. In order to optimize the process of product grouping, we propose the use of the Affinities Theory and Galois Group Theory, which allows us to know exactly which products are stout at determined characteristics with a significant level of confidence.

In the application of the methodology, decision makers of the municipality of Morelia – México, need to group different products, in this case highlights of the city to optimize the visit of a specific profile of tourist. By grouping the most affine places of interest of the city, decision makers can choose from different strategies to maximize the experience a visitor may have.

In order to assess the specific challenge that the municipality of Morelia – México faces we need to establish in one hand the products that will be offered to the visitor and in the other the variables that will be held to evaluate the affinity in which the products relate to each other. We now follow the steps specified in (Gil Lafuente, 2002).

### *Establishing the Products*

The municipality of Morelia – México has a list of 13 highlights, which are considered the prime touristic attractions of the city. We proceed to name them in the following table.

Table 1. Touristic Products

| | |
|---|---|
| a | Planetario |
| b | Palacio Clavijero |
| c | Catedral |
| d | Museo de Sitio Casa de Morelos |
| e | Jardín Villalongín |
| f | Monumento a José María Morelos |
| g | Callejón del Romance |
| h | Zoológico |
| i | Estadio Morelos |
| j | Teatro Melchor Ocampo |
| k | Centro Comercial Espacio las Américas |
| l | Museo Regional Michoacano |
| m | Bosque Cuauhtémoc |

Source: Elaborated from municipal touristic records.

***Formulating the Variables***

A list of characteristics of the places of interest was asked to be given. Each characteristic, singularity and peculiarity will serve as a basis for the creation of affinities for the whole group of products. The list was made by 10 assessors of the tourism office in the city. Their opinions were focused on the main characteristics that a visitor of the city seeks. Once the head of the department approved the list, we proceed to name it.

(A) Historic: the level in which the highlight represents historical facts or events that occurred in the city.

(B) Representative: the level in which the highlight remains on the memory of a visitor and serves for elucidating the city.

(C) Commercial: the level in which that specific highlight allows a visitor to generate economic spillover.

(D) Environmental: the level in which that specific highlight has green areas, and in general is green friendly.

(E) Location: the distance a specific highlight has from the city's' geographical center.

(F) Amenities: the level in which that specific highlight entertains the visitor, expositions, performances and cultural activities are some of the amenities included.

It is important to mention that the variables / characteristics included in the model are not exhaustive and have been treated with the same level of importance; we are currently working on further investigation, in which the nature of the variables and the importance of them affect, and apply certain weights in the model.

***Grouping by Affinities***

The first step to conduct the process of grouping is to generate fuzzy subsets, valuating the different products due to their characteristics, singularities and peculiarities in the next way:

$$\underset{\sim}{i} = \begin{array}{c} A \\ \boxed{\mu_A{}^{(i)}} \end{array} \begin{array}{c} B \\ \boxed{\mu_B{}^{(i)}} \end{array} \cdots \begin{array}{c} N \\ \boxed{\mu_N{}^{(i)}} \end{array}$$

$$i = a, b, c, \dots, m$$

$$\mu_j{}^{(i)} \in [0,1]\,, j = A, B, \dots, N$$

Each product due to its inherent characteristics will be evaluated thru a linguistic tag between 0 and

1 in which:

Table 2. Evaluation of Variables

| | |
|---|---|
| 1 | Excellent performance |
| 0.9 | Great performance |
| 0.8 | Really good performance |
| 0.7 | Good performance |
| 0.6 | Rather a better tan a poor performance |
| 0.5 | Nor a good or poor performance |
| 0.4 | Rather a poor tan a good performance |
| 0.3 | Poor performance |
| 0.2 | Really poor performance |
| 0.1 | Worst performance |
| 0 | Disastrous performance |

Source: Self elaborated.

In our case we have:

$$\underset{\sim}{a} = \begin{array}{|c|c|c|c|c|c|} \multicolumn{1}{c}{A} & \multicolumn{1}{c}{B} & \multicolumn{1}{c}{C} & \multicolumn{1}{c}{D} & \multicolumn{1}{c}{E} & \multicolumn{1}{c}{F} \\ \hline .3 & .7 & .4 & .6 & .4 & .8 \\ \hline \end{array}$$

$$\underset{\sim}{b} = \begin{array}{|c|c|c|c|c|c|} \multicolumn{1}{c}{A} & \multicolumn{1}{c}{B} & \multicolumn{1}{c}{C} & \multicolumn{1}{c}{D} & \multicolumn{1}{c}{E} & \multicolumn{1}{c}{F} \\ \hline .7 & .8 & .2 & .3 & .9 & .8 \\ \hline \end{array}$$

$$\underset{\sim}{c} = \begin{array}{|c|c|c|c|c|c|} \multicolumn{1}{c}{A} & \multicolumn{1}{c}{B} & \multicolumn{1}{c}{C} & \multicolumn{1}{c}{D} & \multicolumn{1}{c}{E} & \multicolumn{1}{c}{F} \\ \hline .8 & 1 & .3 & .7 & 1 & .6 \\ \hline \end{array}$$

$$\underset{\sim}{d} = \begin{array}{|c|c|c|c|c|c|} \multicolumn{1}{c}{A} & \multicolumn{1}{c}{B} & \multicolumn{1}{c}{C} & \multicolumn{1}{c}{D} & \multicolumn{1}{c}{E} & \multicolumn{1}{c}{F} \\ \hline 1 & 1 & .2 & .5 & .8 & .9 \\ \hline \end{array}$$

$$\underset{\sim}{e} = \begin{array}{|c|c|c|c|c|c|} \multicolumn{1}{c}{A} & \multicolumn{1}{c}{B} & \multicolumn{1}{c}{C} & \multicolumn{1}{c}{D} & \multicolumn{1}{c}{E} & \multicolumn{1}{c}{F} \\ \hline .5 & .8 & .3 & .8 & .7 & .3 \\ \hline \end{array}$$

$$\underset{\sim}{f} = \begin{array}{|c|c|c|c|c|c|} \multicolumn{1}{c}{A} & \multicolumn{1}{c}{B} & \multicolumn{1}{c}{C} & \multicolumn{1}{c}{D} & \multicolumn{1}{c}{E} & \multicolumn{1}{c}{F} \\ \hline .8 & .9 & .1 & .7 & .7 & .2 \\ \hline \end{array}$$

$$\underset{\sim}{g} = \begin{array}{|c|c|c|c|c|c|} \multicolumn{1}{c}{A} & \multicolumn{1}{c}{B} & \multicolumn{1}{c}{C} & \multicolumn{1}{c}{D} & \multicolumn{1}{c}{E} & \multicolumn{1}{c}{F} \\ \hline .6 & .6 & .6 & .8 & .7 & .7 \\ \hline \end{array}$$

$$\underset{\sim}{h} = \begin{array}{|c|c|c|c|c|c|} \multicolumn{1}{c}{A} & \multicolumn{1}{c}{B} & \multicolumn{1}{c}{C} & \multicolumn{1}{c}{D} & \multicolumn{1}{c}{E} & \multicolumn{1}{c}{F} \\ \hline .2 & .7 & .5 & 1 & .5 & .9 \\ \hline \end{array}$$

$$\underset{\sim}{i} = \begin{array}{|c|c|c|c|c|c|} \multicolumn{1}{c}{A} & \multicolumn{1}{c}{B} & \multicolumn{1}{c}{C} & \multicolumn{1}{c}{D} & \multicolumn{1}{c}{E} & \multicolumn{1}{c}{F} \\ \hline .1 & .8 & .4 & .5 & .3 & .8 \\ \hline \end{array}$$

$$\underset{\sim}{j} = \begin{array}{|c|c|c|c|c|c|} \multicolumn{1}{c}{A} & \multicolumn{1}{c}{B} & \multicolumn{1}{c}{C} & \multicolumn{1}{c}{D} & \multicolumn{1}{c}{E} & \multicolumn{1}{c}{F} \\ \hline .7 & .6 & .2 & .3 & .9 & .8 \\ \hline \end{array}$$

$$\underset{\sim}{k} = \begin{array}{|c|c|c|c|c|c|} \multicolumn{1}{c}{A} & \multicolumn{1}{c}{B} & \multicolumn{1}{c}{C} & \multicolumn{1}{c}{D} & \multicolumn{1}{c}{E} & \multicolumn{1}{c}{F} \\ \hline 0 & .5 & 1 & .2 & .4 & .7 \\ \hline \end{array}$$

$$\underset{\sim}{l} = \begin{array}{|c|c|c|c|c|c|} \multicolumn{1}{c}{A} & \multicolumn{1}{c}{B} & \multicolumn{1}{c}{C} & \multicolumn{1}{c}{D} & \multicolumn{1}{c}{E} & \multicolumn{1}{c}{F} \\ \hline 1 & .6 & .5 & .4 & .9 & .9 \\ \hline \end{array}$$

$$\underset{\sim}{m} = \begin{array}{|c|c|c|c|c|c|} \multicolumn{1}{c}{A} & \multicolumn{1}{c}{B} & \multicolumn{1}{c}{C} & \multicolumn{1}{c}{D} & \multicolumn{1}{c}{E} & \multicolumn{1}{c}{F} \\ \hline .5 & .5 & .4 & .9 & .8 & .7 \\ \hline \end{array}$$

With this information we generate a fuzzy matrix comprehended by:

$$
\begin{bmatrix} R \\ \sim \end{bmatrix} = \begin{array}{c} \\ a \\ b \\ \\ m \end{array} \begin{array}{cccc} A & B & & N \\ \boxed{\mu_A{}^{(a)}} & \boxed{\mu_B{}^{(a)}} & \cdots & \boxed{\mu_N{}^{(a)}} \\ \boxed{\mu_A{}^{(b)}} & \boxed{\mu_A{}^{(b)}} & \cdots & \boxed{\mu_N{}^{(b)}} \\ & \cdots & \cdots & \cdots \\ \boxed{\mu_A{}^{(m)}} & \boxed{\mu_A{}^{(m)}} & \cdots & \boxed{\mu_N{}^{(m)}} \end{array}
$$

In our case:

$\begin{bmatrix} R \\ \sim \end{bmatrix} =$

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| a | .3 | .7 | .4 | .6 | .4 | .8 |
| b | .7 | .8 | .2 | .3 | .9 | .8 |
| c | .8 | 1 | .3 | .7 | 1 | .6 |
| d | 1 | 1 | .2 | .5 | .8 | .9 |
| e | .5 | .8 | .3 | .8 | .7 | .3 |
| f | .8 | .9 | .1 | .7 | .7 | .2 |
| g | .6 | .6 | .6 | .8 | .7 | .7 |
| h | .2 | .7 | .5 | 1 | .5 | .9 |
| i | .1 | .8 | .4 | .5 | .3 | .8 |
| j | .7 | .6 | .2 | .3 | .9 | .8 |
| k | 0 | .5 | 1 | .2 | .4 | .7 |
| l | 1 | .6 | .5 | .4 | .9 | .9 |
| m | .5 | .5 | .4 | .9 | .8 | .7 |

Once this information has been established and accepted, the decision maker must make a choice concerning the desired level of homogeneity that the groups of highlights may have as for their specific characteristics, qualities and peculiarities. So for each characteristic we will establish:

$$0 \leq \theta_j \leq 1 \,, j = A, B, \dots, N$$

In our case the decision maker defined $\theta$ as:

$$\theta_A = 0.8, \; \theta_B = 0.8, \theta_C = 0.5, \theta_D = 0.3, \theta_E = 0.8, \theta_F = 0.6$$

Once the values of $\theta_j$ have been established, the valuations of each column of characteristics will be compared. If the valuation given to the specific characteristic is equal or superior to the desired level of homogeneity then the valuation is substituted with a 1, in the contrary 0. Specifically:

$$\mu_j{}^{(i)} \geq \theta_j \quad , \; \beta_j{}^{(i)} = 1$$

$$\mu_j^{(i)} < \theta_j \quad , \ \beta_j^{(i)} = 0 \ ,$$

$$i = a, b, \dots, m$$

$$j = A, B, \dots, N$$

By performing this action we will get a new matrix, in which the slots will only have 0 or 1. In our case

$[B] =$

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| a | | | | 1 | | 1 |
| b | | 1 | | 1 | 1 | 1 |
| c | 1 | 1 | | 1 | 1 | 1 |
| d | 1 | 1 | | 1 | 1 | 1 |
| e | | 1 | | 1 | | |
| f | 1 | 1 | | 1 | | |
| g | | | 1 | 1 | | 1 |
| h | | | 1 | 1 | | 1 |
| i | | 1 | | 1 | | 1 |
| j | | | | 1 | 1 | 1 |
| k | | | 1 | | | 1 |
| l | 1 | | 1 | 1 | 1 | 1 |
| m | | | | 1 | 1 | 1 |

***Maximum inverse correspondence algorithm***

In order to find the most affine elements of the highlights of the city, we will follow the theory of affinities, specifically the maximum inverse correspondence algorithm (Gil Aluja, 1999). Studies and applications of this algorithm in economic and business sectors have led to excellent results while dealing with uncertain conditions.

1) From the conjunct of highlights and characteristics choose the one conjunct that presents the fewer elements. In our case:

$$\{A, B, C, D, E, F\}$$

2) Create the "power set", which represents all the possible combinations of the conjunct with the fewer elements. In our case:

{∅, A, B, C, D, E, F, AB, AC, AD, AE, AF, BC, BD, BE, BF, CD, CE, CF, DE, DF, EF, ABC, ABD,

ABE, ABF, ACD, ACE, ACF, ADE, ADF, AEF, BCD, BCE, BCF, BDE, BDF, BEF, CDE, CDF, CEF, DEF,

ABCD, ABCE, ABCF, ABDE, ABDF, ABEF, ACDE, ACDF, ACEF, ADEF, BCDE, BCDF, BCEF, BDEF, CDEF,

ABCDE, ABCDF, ABCEF, ABDEF, ACDEF, BCDEF, ABCDEF}

3) For each element of the "power set" include the corresponding elements of the conjunct that hasn't been chosen for having a greater number of elements. The so called "connection to the right". In our case:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ∅ | → | acbdefghijklm | ABC | → | ∅ | ABCF | → | ∅ |
| A | → | cdfl | ABD | → | cdf | ABDE | → | cd |
| B | → | bcdefi | ABE | → | cd | ABDF | → | cd |
| C | → | ghkl | ABF | → | cd | ABEF | → | cd |
| D | → | abcdefghijlm | ACD | → | l | ACDE | → | l |
| E | → | bcdjlm | ACE | → | l | ACDF | → | l |
| F | → | abcdghijklm | ACF | → | l | ACEF | → | l |
| AB | → | cdf | ADE | → | cdl | ADEF | → | cdl |
| AC | → | l | ADF | → | cdl | BCDE | → | ∅ |
| AD | → | cdfl | AEF | → | cdl | BCDF | → | ∅ |
| AE | → | cdl | BCD | → | ∅ | BCEF | → | ∅ |
| AF | → | cdl | BCE | → | ∅ | BDEF | → | bcd |
| BC | → | ∅ | BCF | → | ∅ | CDEF | → | l |
| BD | → | bcdefi | BDE | → | bcd | ABCDE | → | ∅ |
| BE | → | bcd | BDF | → | bcdi | ABCDF | → | ∅ |
| BF | → | bcdi | BEF | → | bcd | ABCEF | → | ∅ |
| CD | → | ghl | CDE | → | l | ABDEF | → | cd |
| CE | → | l | CDF | → | ghl | ACDEF | → | l |
| CF | → | ghkl | CEF | → | l | BCDEF | → | ∅ |
| DE | → | bcdjlm | DEF | → | bcdjlm | ABCDEF | → | ∅ |
| DF | → | abcdghijlm | ABCD | → | ∅ | | | |
| EF | → | bcdjlm | ABCE | → | ∅ | | | |

4) We choose, from every non-void conjunct of the "connection to the right" the corresponding conjunct of the "power set", which possess the greater number of elements. In our case:

| | | |
|---|---|---|
| ∅ | → | ABCDEF |
| cd | → | ABDEF |
| l | → | ACDEF |
| cdl | → | ADEF |
| bcd | → | BDEF |
| cdf | → | ABD |
| bcdi | → | BDF |

| | | |
|---|---|---|
| ghl | → | CDF |
| bcdjlm | → | DEF |
| cdfl | → | AD |
| bcdefi | → | BD |
| ghkl | → | CF |
| abcdghijlm | → | DF |
| abcdefghijlm | → | D |
| abcdghijklm | → | F |
| abcdefghijklm | → | Ø |

5) At this point we have found the maximum number of relations, named affinities. The algorithm applied allowed in an unambiguous method to create the biggest amount of groups, due to the desired homogeneity level. In our case the highlights of the city can be grouped in any of the specified conjuncts due to the characteristics, qualities and peculiarities they present.

The relations found between both conjuncts create themselves a Galois Lattice, which allows demonstrating in an ordered way the homogeneous groups as well as the perfect structuration of the elements.

***Galois group theory and Galois lattices***

Galois Theory is a connection between the field theory and the group theory. Certain problems in field theory can be reduced to group theory using Galois Theory. This allows us understanding the problems easier and solving them in a simpler way. In the beginning, Galois used permutation groups to explain how the various roots of a given polynomial equation were related to each other, Edwards (1984).

Galois Theory is based on a remarkable correspondence between subgroups of the Galois group of an extension E/F and intermediate fields between E and F.

If G = Gal (E/F) is supposed to be the Galois group of the extension E/F. If H is a subgroup of G, the fixed field of H is the set of elements fixed by every auto-morphism in H, that is:

$F(H) = \{x \in E : \sigma(x) = x \text{ } for \text{ } every \text{ } \sigma \in H\}.$

If $K$ is an intermediate field, that is, $F \leq K \leq E$ define:

$G(K) = Gal(E/K) = \{\sigma \in G : \sigma(x) = x \text{ } for \text{ } every \text{ } x \in K\}.$

In other words fixing group of K for G (K), since G (K) is the group of auto-morphisms of E that leave K fixed. Galois Theory is about the relation between fixed fields and fixing groups, see Edwards (1984); Artin (1998).

***Definitions of the theory***

Following the definition of Keropyan & Gil-Lafuente (2013):

*Definition 1.* A lattice is a partially ordered set in which two any elements have a least upper bound (LUB) and a greatest lower bound (GLB). A complete lattice is a lattice where any set has a LUB and a GLB.

*Definition 2.* A context K is a triple $(O, F, \zeta)$where O is a set of objects; F is a set of attributes and $\zeta$ is a mapping from $O \times F$ into $\{0, 1\}$.

*Definition 3.* Given a context $K = (O, F, \zeta)$ let us define two mappings from $P(O)$ into $P(F)$ and from $P(F)$ into $P(O)$ using the same notation 0 by the formula:

$$\forall A \subset O, A' = \{f \in F | \forall o \in A, \zeta(o, f) = 1\}$$

$$\forall B \subset F, B' = \{o \in O | \forall o \in B, \zeta(o, f) = 1\}$$

$A'$ Is called the dual of A, similarly $B'$ is called the dual of B.

*Definition 4.* Given a context $K = (O, F, \zeta)$, the pair $C = (A, B)$ is called a concept of K if and only if $A' = B$ and $B' = A$.

*Definition 5.* A is called the extent of the concept C and B is called its intent.

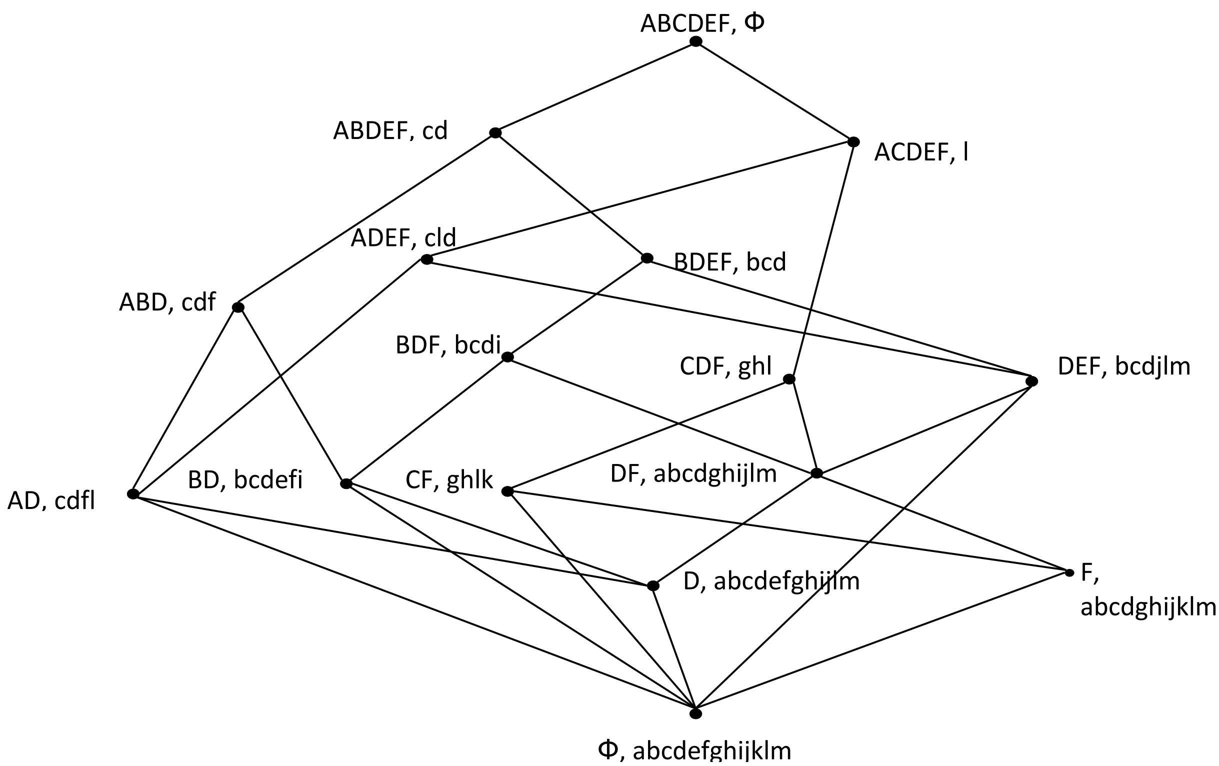
This is denoted by $A = extent\ (C)$ and $B = intent\ (C)$.

Considering an order relationship defined through inclusion of intents, one may define a Galois lattice or concept lattice.

*Definition 6.* The complete lattice $L(K)$ of concepts of the context $K$ is called (general) Galois lattice or concept lattice.

In our case we represent the Galois Lattice as follows:

Figure 1.Galois Lattice for the City's Highlights Case




Source: Self elaborated.

## RESULTS AND DISCUSSION

The algorithm finishes when we obtain Galois Lattice, the figure represents in an ordered and systematic way, not only the total number of affinities that exist between the highlights of the city and the variables that comprise them, but it interconnects them in a coherent structure.

The decision maker can now opt for diverse combinations that could enhance the current schemes of touristic plans. In a first instance we can see that if we wanted to group all the tourist attractions it would be impossible because each highlight has different valuation on their specific characteristics, qualities and singularities. As the levels advance we can visualize how the groups establish, in a first level highlights a, b, c, d, e, f, g, h, I, j, l and m all share the characteristic D or Environmental, in a similar way a, b, c, d, g, h, I, j, k, l and m all share characteristic F or Amenities. Perhaps the following levels could be more useful to the decision maker, since the amount of characteristics grow but the quantity of places of interest decrease. For example highlights c, d, f and i possess characteristics A and D, Historic and Environmental. The maximum number of characteristics possessed by a group of highlights is found in the top of Galois Lattice representation, where c and d present singularities A, B, D, E and F.

The maximum number of groups have been presented in an ordered and structured way, the decision maker has now the possibility to generate structured plans following the levels of the Lattice, if the plan requires groups of highlights which present A, D, E and F characteristics, then c, l, and d places are the most affine and could create better synergies. Following the same idea, if the plans require D, E, and F characteristics then b, c, d, j, l and m are the most affine highlights to generate common strategies.

This result is highly interconnected with the level of homogeneity chosen by the experts; in this case that level was the result of a specific profile of tourist. The decision maker could generate different scenarios, applying diverse combinations in order to get a full map of groups, depending on the various profiles of visitors that the city receives.

## CONCLUSIONS

We propose an original group-based model methodology that relies on the comparison between determined variables collected by the inherent characteristics of different products in order to create positive synergies between them. The proposed model is originated on the basic principles of Galois group theory, this process allows grouping different products with a certain level of significance, detect the level in which those groups could create synergies, and select which of the inherent characteristics of the products could be enhanced due to the specific needs and requirements of the decision maker.

The present work tries to shed a light in the academic world by offering a robust group based model in which subjective and relative factors are intrinsic for the decision making process. Also this analysis tries to aid decision makers so they can create common policies due to the results of the grouping processes.

Further research needs to be conducted, at a first instance, study the nature of the variables stated to know whether they need to be weighted, conduct tests to know if this weight plays a significant role on the results obtained and also apply the model in specific conditions. The model we present can be applied to different circumstances; we would like to encourage research on similar areas since it may allow optimizing the process of grouping of products under subjective and uncertain conditions.

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